

Power Allocation for Hybrid Overlay/Underlay CRN

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Abstract

In this paper, an optimal power allocation technique for hybrid CRN with real-time and non-real-time services is employed. The proposed method maximizes the rate requirement for NRT users, while maintaining the minimum rate requirement for RT users by opportunistically switching between underlay mode and overlay mode. The interference constraint in this type of network is a challenge that, the interference should be kept below a predefined threshold value; this makes the optimization problem more complex. To overcome the resulting computational hurdle polyhedral approximation of second order cone is considered to optimize the power and sub carrier allocation for NRT users. The same can be used to RT user with minimum threshold for data rate is imposed. A near optimal solution can be obtained based on the Lagrangian dual.

Keywords—Lagrangian dual, Polyhedral .

I. INTRODUCTION

An increasing development and popularity of wireless communication turned the limited spectrum into a scarce resource. In fact, recent studies by FCC have shown that the utilization of licensed spectrum varies from 15% to 85%. The key technology towards efficient spectrum usage is cognitive radio. CR allows unlicensed users (SU) to access the licensed band under the condition of protecting the licensed users (PU) from harmful interference.

Three approaches have been developed for CR .So far regarding the way a secondary user accesses the licensed spectrum. (i) Spectrum overlay (Opportunistic spectrum access) where the secondary user transmit only when a frequency band is detected to be idle. (ii) Spectrum underlay (Spectrum sharing) where the secondary user coexist with the primary user and apply an interference constraint to ensure the quality of service of the primary network. (iii) Hybrid underlay/overlay (Sensing based spectrum sharing) where secondary user first sense the status of the PU (idle/active) and then select the appropriate sensing scheme. If the PU is idle SU select the spectrum overlay mode and transmit with maximum power budget for a higher data rate. If the PU is active SU select the spectrum underlay mode and transmit with lower power budget for a higher data rate. In such hybrid overlay/underlay mode the SU transmit both in idle and busy bands. It improves the secondary network throughput by maintaining the harmful interference to primary network and also guarantees the QoS of primary network. Heterogeneous services for both real-time and non-real-time services exist in CRN. Non Real time services has minimum threshold data rate for transmission. Real-time services has

minimum data rate and minimum interference to PU for transmission.

A. Related Work

In [1] Power allocation in multiuser orthogonal frequency division multiplexing based cognitive radio network is considered overlay/underlay mode not considered. In [2] Resource allocation in multiband cognitive radio for underlay/overlay mode spectrum sharing scheme, allowing the secondary user to adapt its way of accessing the licensed spectrum to the status of primary user. Resource allocation for real-time and non-real-time services is not employed. In [3] consider the channel uncertainty of link between primary user and secondary user, also proposed a method to optimization problem which yields better results for non-real-time users resource allocation. The problem of resource allocation for real-time user not addressed.

II. SYSTEM MODEL AND PROBLEM STATEMENT

Consider a CR-BS allocating resources to K CR-MS. The channel gain $h_k^{(n)} \geq 0$ between CRBS and the k^{th} CR-MS for each $k \in K; K = \{1, 2, \dots, K\}$ and each subcarrier $n \in N; N = \{1, 2, \dots, N\}$. During sensing phase the CR-BS detected the PU is present. Let $p^{(n)}$ denote the transmit power loaded on subcarrier n and $p_{\max}^{(n)}$ the maximum transmit power allowed on subcarrier n . Vector p and p_{\max} collect $\{p^{(n)}\}$ and $\{p_{\max}^{(n)}\}$ respectively. The received signal for user k on the subcarrier n is given as

$$y_k^{(n)} = \sqrt{h_k^{(n)} p^{(n)} x_k^{(n)}} + v_k^{(n)} \quad (1)$$

Where $x_k^{(n)}$ is the unit average power input signal on subcarrier n . $v_k^{(n)}$ complex Gaussian noise with mean 0 and unit variance.

A. Problem Statement

A weighted sum rate maximization problem is formulated for a CR-BS that transmits to a set of CR-MS, while respecting a strict interference constraint converted to be second order cone constraint which is convex even multiple users are considered. Optimal allocation of spectrum for SU transmit powers in order to maximize the SU throughput under interference and channel uncertainty while preserving the QOS of primary system. $g^{(n)} \geq 0$ denote the channel gain from CR-BS to the primary receiver on the subcarrier n . $\{g^{(n)}\}$ are assumed to be uncertain. The uncertainty in $g := [g^{(1)} g^{(2)} \dots g^{(N)}]^T$ vector can be captured by an ellipsoidal uncertainty region given by

$$G := \left\{ \bar{g} + \Delta g : \Delta g^T C_g^{-1} \Delta g \leq \Omega^2 \right\} \quad (2)$$

Where \bar{g} is the nominal value of g .

Δg the deviation from the nominal value. C_g a symmetric positive definite matrix and $\Omega \geq 0$ a given constant. The optimization problem is given as

$$0 \leq p \leq P_{\max}^{max_{k \in K} N} \sum_{n=1}^N w_{k(n)} I_{k(n)}^{(n)} (h_k^{(n)} p^{(n)}) \quad (P1)$$

$$\text{Subject to } \sum_{n=1}^N p^{(n)} \leq P_{\max} \quad (4)$$

P1 is non convex due to the combinational assignment of users on each subcarrier. When an OFDMA RA problem has a separable structure in which the Lagrangian dual can be decomposed into per subcarrier problems it can be shown that the duality gap vanishes as the number of subcarriers increases.

III. MODIFIED POLYHEDRAL

APPROXIMATION OF SECOND ORDER CONES

To convert the non-convex problem to convex problem the second order Lorentz cone is used $L^N := \{(y_0, Y) \in R \times R^N : \|Y\| \leq y_0\}$ (5)

Admits a polyhedral approximation of accuracy δ . L^N in the $(N+1)$ dimensional space is bound to have its number of facets growing exponentially with the dimension N . This increase the number of inequalities. The key idea to reduce the number of inequalities by lifting the polyhedral to higher order space. The first step is to decompose the

$(N+1)$ dimensional Lorentz cone L^N to a number of 3-dimensional Lorentz cone. Decomposing further the $(N/2 + 1)$ dimensional Lorentz cone by applying the same idea repeatedly. The remaining task is to approximate L^2 using a polynomial number of variables and constraints.

The set of points $(y_0, y_1, y_2) = (\alpha_{q+1}, \alpha_0, \beta_0)$ satisfying the following set of linear constraints in a δ relaxation of L^2 .

$$\alpha_{i+1} = \alpha_i \cos\left(\frac{\pi}{2^i}\right) + \beta_i \sin\left(\frac{\pi}{2^i}\right), i = 0, 1, \dots, q \quad (6)$$

$$\beta_{i+1} \leq \left| \beta_i \cos\left(\frac{\pi}{2^i}\right) - \alpha_i \sin\left(\frac{\pi}{2^i}\right) \right|, i = 0, 1, \dots, q-1 \quad (7)$$

Where $\alpha := [\alpha_1, \alpha_2, \dots, \alpha_q]^T$, $\beta := [\beta_1, \beta_2, \dots, \beta_q]^T$ are extra variables introduced to lift the approximation to a higher dimensional space. Thus $2q$ extra variables have been introduced to form $(q+1)$ equality constraint and $2q$ inequality constraint. One can further reduce the number of variables and constraints by eliminating α and β_q using the equalities.

IV. POWER ALLOCATION ALGORITHM

A. Algorithm based on Lagrangian Dual

Optimization problem is a conservative substitute for P1.

$$P_{\max}^{max_{k \in K} N} \sum_{n=1}^N w_{k(n)} I_{k(n)}^{(n)} (h_k^{(n)} p^{(n)}) \quad (P2)$$

$$\text{Subject to } \sum_{n=1}^N p^{(n)} \leq P_{\max} \quad (8)$$

$$0 \leq p^{(n)} \leq p_{\max}^{(n)} \quad n=1, 2, \dots, N \quad (9)$$

$$Ap + Bq \leq b \quad (10)$$

Problem P2 is again non-convex. However it can be shown that the duality gap vanishes asymptotically as $N \rightarrow \infty$. Therefore P2 can be solved efficiently using the dual method. Introducing dual variables $\lambda \geq 0$ and $\mu := [\mu_1, \mu_2, \dots, \mu_{n_c}]^T \geq 0$.

The partial Lagrangian is given as

$$L(p, q) = \sum_{n=1}^N w_{k(n)} I_{k(n)}^{(n)} (h_k^{(n)} p^{(n)}) - \lambda \left(\sum_{n=1}^N p^{(n)} - P_{\max} \right) - \mu^T (Ap + Bq - b) \quad (11)$$

$$= \sum_{n=1}^N \{w_{k(n)} I_{k(n)}^{(n)} (h_k^{(n)} p^{(n)}) - (\lambda + \mu^T A(:,n)) p^{(n)}\} + \lambda p_{\max} - \mu^T Bq + \mu^T b \quad (12)$$

Where $A(:,n)$ denotes the n -th column of matrix A . The dual function is given by

$$D(\lambda, \mu) = \sup_{0 \leq p \leq p_{\max}, k \in K^N} L(p, q) \quad (13)$$

$$= \begin{cases} \sum_{n=1}^N L_{n(p^{(n)}, k(n)) + \lambda p_{\max} + \mu^T b}, & \text{if } B^T \mu = 0 \\ \infty & \text{otherwise} \end{cases} \quad (14)$$

and thus the dual optimization problem goes to

$$\inf_{\lambda, \mu} D(\lambda, \mu) \quad (15)$$

$$\text{Subject to } \lambda \geq 0, \mu \geq 0, B^T \mu = 0$$

(16)

The optimal user allocation K^* and power loading P^* are given as

$$k^{*(n)} \in \arg \max_k L_n(p^{*(n)}[k], k), \quad n=1, 2, \dots, N \quad (17)$$

$$P^{*(n)} = p^{*(n)}[k^{*(n)}] \quad n=1, 2, \dots, N \quad (18)$$

The optimal solution can be obtained via iterative optimization methods for non-differentiable objectives, such as the ellipsoid method. To examine the complexity of the algorithm the overall complexity can be characterized by multiplying the complexity per iterations by the number of iterations needed. The number of iterations needed for the ellipsoid method to

converge grows as the square of the number of optimization variables.

B. Suboptimal Algorithm

Performance of the proposed near-optimal scheme, a simple suboptimal algorithm based on alternating minimization is also considered. The procedure can be described in pseudo code as follows

Step 1: Initialize p , e.g., set $P^{(n)} = P_{\max}/N$ for $n=1, 2, \dots, N$

Step 2: Set $k(n) = \arg \max_k \omega_k I_k^n (h_k^{(n)} p^{(n)})$ for $n=1, 2, \dots, N$

Step 3: With k fixed solve (p1) only over P using convex optimization techniques

Step 4: If the objective is not increased, stop; otherwise, go to step 2.

V. SIMULATION RESULTS

The proposed algorithm is verified via simulation results.

A. Simulation Setup

A Rayleigh α -faded, 4 path channel is simulated. The pathloss exponent is set to $\alpha = 2$. Assuming Gaussian distributed g , We set $\Omega = Q^{-1}(\epsilon)$ for small $\epsilon > 0$. The values of $p_{\max} = 10^2$ and $I_{\max} = 1$ were used with unit power channel coefficient and additive noise. When $K = 1$ the original problem is convex, which can be easily solved using generic convex optimization software for optimal power allocation subcarriers. In the multiuser case with $K > 1$, problem 1 becomes hard due to the combinatorial search for optimal subcarrier assignment.

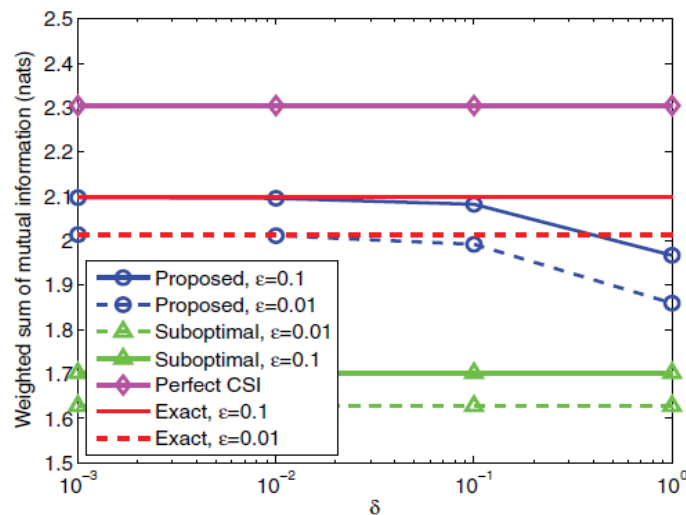


Fig 1 Weighted sum rate when K=2.

Fig 1 shows the case of $K=2$ users, with user 2 located four times farther than user 1 from the CR-BS. Equal weights of $\omega_1 = \omega_2 = 0.5$ were used. User 1 employed a BPSK constellation and user 2 QPSK and the number of subcarriers $N=8$.

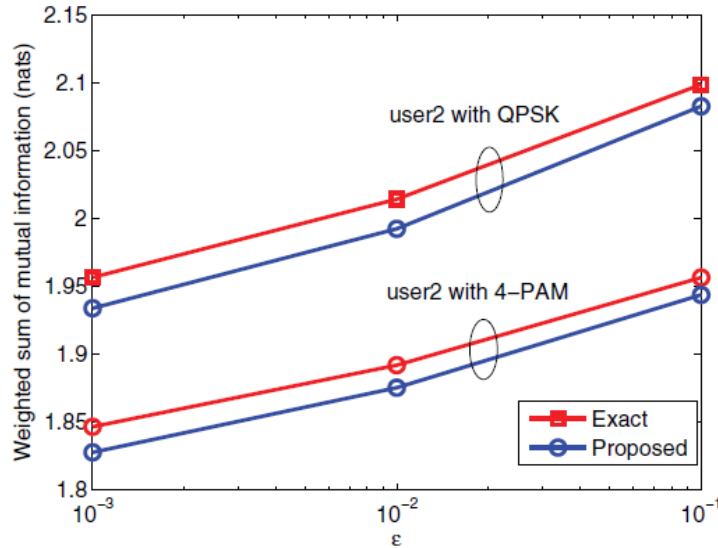


Fig 2 Weighted sum rate Vs ϵ

Fig 2 shows the weighted sum rates for the same setup as used for fig 1 but with user 2 employing QPSK or 4PAM modulations. The value of $\delta=0.1$ was used and the values of ϵ were varied.

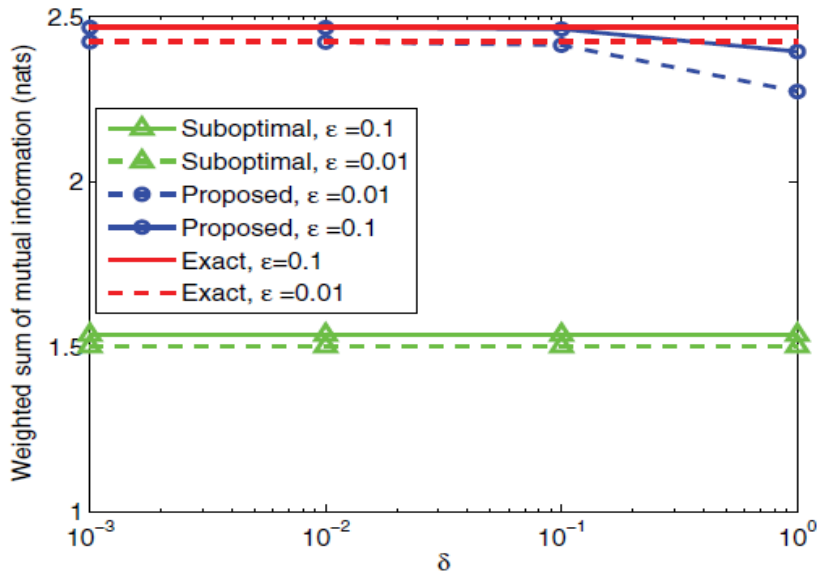


Fig 3 Weighted sum rate when $K=3$

Fig 3 shows the case with $K=3$ CR users for $\epsilon=0.1$ and $\epsilon=0.01$. The distance of users 2 and 3 from the CR-BS were 4 times of user 1, and the input constellation for user 1,2,3 were set to BPSK,4PAM and QPSK respectively. Equal weights were used. The value of $I_{max}=10$ and $N=8$ subcarriers were employed.

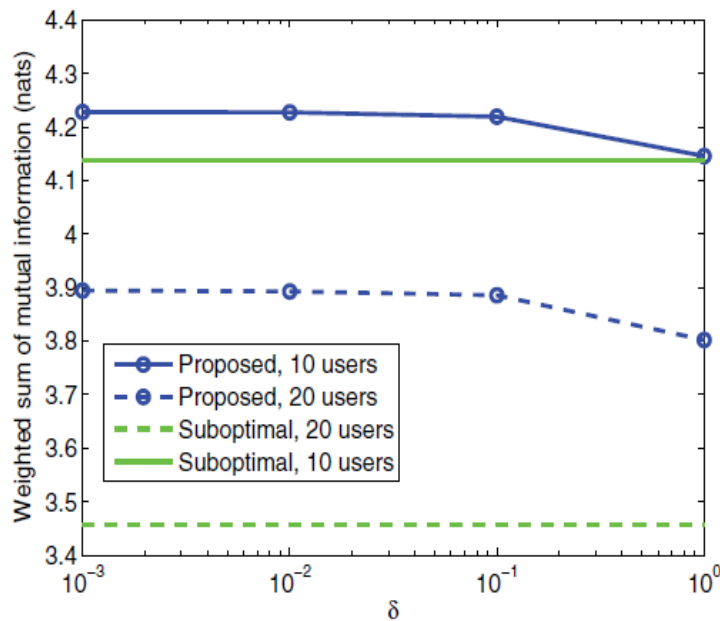


Fig 2 Weighted sum rate When K=10 and K=20.

The proposed algorithm was tested for larger values of K. Fig 4 shows the case with K=10 and K=20 where N=32 and $\varepsilon=0.1$ were used.

VI. CONCLUSION

The weighted sum rate maximization problem was formulated for a CR system employing OFDMA. Due to the uncertainty present in the CR to PU channel, a robust interference constraint with an ellipsoidal uncertainty set was imposed to protect the PU system, which is equivalent to a second order cone constraint. The dual method could then be employed to compose the overall problem into per sub carrier subproblems, which can be easily solved. The overall algorithm efficiently finds the near optimal power loading and sub carrier assignment.

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